ELIZADE UNIVERSITY

ILARA-MOKIN

FACULTY: BASIC AND APPLIED SCIENCES

DEPARTMENT: MATHEMATICS AND COMPUTER SCIENCE

1st **SEMESTER EXAMINATION** 2019 / 2020 **ACADEMIC SESSION**

COURSE CODE: CSC 435 **COURSE TITLE:** Numerical Analysis

COURSE LEADER: Dr. Vincent Akpan **DURATION:** 3 Hours

HOD's SIGNATURE

INSTRUCTION: Answer any **four** (4) questions with one from each PART.
Students are warned that possession of any unauthorized materials in an examination is a serious assessment offence. Students are permitted to use ONLY a scientific calculator.

PART A

1. (a) Distinguish between the following terms:
(i) Interpolation (ii) Curve fitting (iii) Time-Series
(b) Given the following data points,

x	0	2	3
y	7	11	28

Use the Lagrange's method to determine y at $x = 1$.

(c) Determine the parameters a and b so that $f(x) = ae^{bx}$ fits the following data in the least-squares:

x	1.2	2.8	4.3	5.4	6.8	7.9
y	7.5	16.1	38.9	67.0	146.6	266.2

Fit $\ln y_i$ and compute the standard deviation.

2. (a) (i) What is least-squares fit?
(ii) What is linear regression?
(b) Suppose that following data points lie on a polynomial.

x	-2	1	4	-1	3	-4
y	-1	2	59	4	24	-53

Determine the degree of this polynomial by constructing the divided difference table.
(c) Use natural cubic spline to determine y at $x = 1.5$. The data points are:

x	1	2	3	4	5
y	0	1	0	1	0

PART B

3. (a) The behaviour of a fairly large class of discrete-time systems can be modeled by the following general mathematical formula:

$$A(z^{-1})Y(k) = z^{-d}\frac{B(z^{-1})}{F(z^{-1})}U(k) + \frac{C(z^{-1})}{D(z^{-1})}e(k)$$

(where all symbols have their usual meaning).
Using the above equation, deduce the mathematical model structure that corresponds to the:
(i) AutoRegressive with eXogenous inputs (ARX) model.
(ii) AutoRegressive Moving Average with eXogenous inputs (ARMAX) model.
(iii) Output Error (OE) model.

(b) Using the results in (c) and starting from any known principles, deduce an expression and draw the resulting neural network model structure that corresponds to the:
(i) Neural Network AutoRegressive with eXogenous inputs (NNARX) model.
(ii) Neural Network AutoRegressive Moving Average with eXogenous inputs (NNARMAX) model.
(iii) Neural Network Output Error (NNOE) model.

(c) Assuming that 3 past inputs and outputs are sufficient to model a 5-input 3-output system using a dynamic feedforward neural network autoregressive moving average with exogenous inputs (NNARMAX) model with 15 input-to-hidden layer neurons.
(i) What is the number of output neurons?
(ii) Compute the total number of inputs to the neural network.
(iii) Compute the dimension of the input-to-hidden layer weight.
(iv) Compute the dimension of the hidden-to-output layer weight.

4. (a) Fuzzy logic model comes in two flavours, namely: Mamdani-type and Sugeno-type (also called Takagi-Sugeno-Kang (TKS)).
(i) State where each type of the above fuzzy logic models can find applications.
(ii) Briefly state three advantages each for the two model types.

(b) (i) What is the main argument for the introduction of the Adaptive neural fuzzy inference system (ANFIS).
(ii) Draw the typical architecture of a five-layer ANFIS and state the function of each layer.

(c) A single layer perceptron is initialized with weights $w_1 = 1$ and $w_2 = 2$ with bias $b = -2$ for a simple output classification problem. Given the inputs $u = [0.5, 0.5]$ and the target output as $y = +1$. Assuming that the network has an Heaviside activation function where $F(\cdot) = 1$, if $(\cdot) > 0$ and $F(\cdot) = -1$, otherwise.
(i) What are the final values of the weights and bias?
(ii) In how many iterations does the perceptron output converge to the desired target output?

PART C

5. (a) (i) Distinguish between numerical differentiation and numerical integration.
(ii) State which of numerical differentiation or integration is more efficient. Give at least two reasons to support your answer.

(b) (i) Derive the Newton-Cotes formulas.
(ii) Derive the trapezoidal rule from 5(b)(i) above.
(iii) Derive the composite trapezoidal rule from 5(b)(ii) above.

6. (a) In four–step sequence, write down the Nelder-Mead Algorithm.

(b) Write a MATLAB program that employs the Nelder-Mead Algorithm to minimize a two-variable objective function given as:

$$f(x_1, x_2) = x_1^2 - x_1x_2 - 4x_1 + x_2^2 - x_2$$

(c) Write down the minimum values of x_1 and x_2 as well as the function $f(x_1, x_2)$ value at that points.

PART D

Instruction for Part D: Please copy and paste or using any other method, submit all diagrams as Microsoft Word document together with the folder containing all the work done in this Part into a USB flash.

7. Given the data set in the folder *Part_D_data*, assuming that of 4 past inputs and outputs are sufficient to model the system represented by the given data set, obtain a nonlinear autoregressive with exogenous input (NARX) model using the following MATLAB/Simulink toolboxes:
   (a) System identification toolbox (Hint: use the data *sys_ident_data_inputs* and *sys_ident_data_outputs*)
      (i) Show the system identification architecture for the model development
      (ii) Show the plot of the validation results
      (iii) Show the plot of the training error or performance index.
   (b) Neural network time series toolbox (Hint: use data *nn_data_inputs* and *nn_data_inputs*)
      (i) Show the neural network architecture
      (ii) Show the plot of the validation results
      (iii) Show the plot of the training error or performance index.
   (c) (i) Compare your results obtained in 7(a) and (b), and state which method gives the best result.
      (ii) Is there any reason why this result should be this way?
8. Fuzzy Rule-Based Logic Brake–Force Control of a robot.

Hint: Two *.fis* files have been included for guidance.

Problem Definition: Given the velocity of a robot and the space between the robot and an obstacle or a leading vehicle, develop a fuzzy system to control the brake force of the vehicle.

Inputs: * Velocity : Suppose that the robot have a top speed of 150 m/h.
* Space : Suppose that the space can take values from 0 to 50 meters.

Output: * Brake-force: Suppose that the brake-force can take values from 0 to 100, where 100 represent the maximum brake-force.

Linguistic Variables: * Velocity → Term Set: {Slow, Medium, Fast}
* Space → Term Set: {Near, Medium, Far}
* Brake-Force → Term Set: {Small, Medium, Big}

Fuzzy Rule Based-Logic Design and Implementation:

(a) ***First case:***

1). IF velocity is slow AND space is far THEN brake-force is small.
2). IF velocity is medium AND space is medium THEN brake-force is medium.
3). IF velocity is fast AND space is medium THEN brake-force is medium.
   (i) From the surface, what can you notice?
   (ii) For the velocity-space regions, what could you deduce about the fuzzy rule-based system?
   (iii) What can you say about the transitions based on the surface views?
   (iv) What do you think could be responsible for the behaviours based on the observations in 8(a)(i) – (a)(iii)?

(b) ***Second case:***

1). IF velocity is slow AND space is near THEN brake-force is medium.
2). IF velocity is slow AND space is medium THEN brake-force is small.
3). IF velocity is slow AND space is far THEN brake-force is small.
4). IF velocity is medium AND space is near THEN brake-force is big.
5). IF velocity is medium AND space is medium THEN brake-force is medium.
6). IF velocity is medium AND space is far THEN brake-force is small.
7). IF velocity is fast AND space is near THEN brake-force is big.
8). IF velocity is fast AND space is medium THEN brake-force is big.
9). IF velocity is fast AND space is far THEN brake-force is medium.
   (i) From the surface, what can you notice?
   (ii) For the velocity-space regions, what could you deduce about the fuzzy rule-based system?
   (iii) What can you say about the transitions based on the surface views?
   (iv) What do you think could be responsible for the behaviours based on the observations in 8(b)(i) – (b)(iii)?

(c) ***Third case:***

1). Change the shape and the parameters of the membership functions to Gaussian, trapezoidal, etc of your choice.
2). Change the AND, implication, aggregation and defuzzification methods.
3). Change the number and the form of the rules in the fuzzy rule bases.
   (i) Can you decide upon the best parameter set of the fuzzy rule-based system?
   (ii) What conclusion can you deduce from the three cases?